ELL CO-OP PLC WIDA/GSE Alignment Project

Mathematics

Number and Operations

Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and having students compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts.

Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation(s) to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties. A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

M(N&O):4:1	**Demonstrates conceptual understanding of rational numbers with respect to: whole numbers** from 0 to 999,999 through equivalency, composition, decomposition, or place value **using models, explanations, or other representations;** and **positive fractional numbers** benchmark fractions: *a/2, a/3, a/4, a/5,/6,/8,* or *a/10,* where *a* is a whole number greater than 0 and less than or equal to the denominator) as a part to whole relationship in are, as, or linear models where the number of parts in the whole are equal to, and a multiple or factor of the denominator; and **decimals** as hundredths within the context of money, or tenths within the context of metric measurements (e.g., 2.3 cm) **using models, explanations, or other representations.**
Level 1 Entering	Matches word cards showing vocabulary of fractional numbers, (e.g., *part, whole, half, quarter, fractions)* with a visual model.
Level 2 Beginning	Identifies fractions or decimals (e.g., identify the written form of a fraction spoken by the teacher).
Level 3 Developing	Writes a fraction or decimal that corresponds to a visual representation (e.g., writes 3/4 next to a picture with three quarters of a shape shaded).
Level 4 Expanding	Translates word phrases into mathematical symbols (e.g., *one half of the class are boys:* student writes *1/2 class = boys*), with a partner.
Level 5 Bridging	Writes a problem using fractions, translates the phrases into mathematical symbols, and solves the problem.

M(N&O):4:2	**Demonstrates understanding of the relative magnitude of numbers** from 9 to 999,999 by ordering or comparing whole numbers; and ordering, comparing,

	or identifying equivalent proper positive fractional numbers; or decimals using models, numbers lines, or explanations.
Level 1 Entering	Identifies and names real numbers and basic mathematical symbols (e.g., decimal point, >, <, +, –, x, ÷, =) by matching the symbols with the written or spoken word.
Level 2 Beginning	Orders and compares numbers using terms such as *more, less, smaller, bigger, equal,* given models or illustrations (e.g., base ten blocks, number lines, pictures).
Level 3 Developing	Describes the relative magnitude of a number, using terms such as *greater than, less than, equal to,* with visual supports (e.g., fraction bars, number lines, graphs).
Level 4 Expanding	Orders and compares numbers using number lines, charts, and graphs, and explains reasoning, using terms such *as equal to, greater than, less than.*
Level 5 Bridging	Orders fractions on a number line; then discusses in a small group why smaller fractions have larger denominators.

M(N&O):4:3	**Demonstrates conceptual understanding of mathematical operations** by describing or illustrating the relationship between repeated subtraction and division (no remainders); the inverse relationship between multiplication and division of whole numbers; or the addition or subtraction of positive fractional numbers with like denominators using models, number lines, or explanations.
Level 1 Entering	Matches the visual representation of a repeated subtraction problem (e.g., *9 – 3 – 3 – 3 = 0*) to the visual representation of the related division problem *(9 divided by 3 equals 3)*.
Level 2 Beginning	Writes and solves a repeated subtraction problem spoken by the teacher (e.g., *9 – 3 – 3 – 3 = 0*); then writes and solves the corresponding division problem *(9 ÷ 3 = 3,),* using manipulatives.
Level 3 Developing	Writes and solves a multiplication problem from the teacher's oral prompt (e.g., teacher states *6 times 2 equals 12*; student writes and solves the problem); student then writes and solves the inverse division problem, using manipulatives if needed.
Level 4 Expanding	Writes a division problem that is the inverse of a multiplication problem shown on a card (e.g., student picks card showing 5 x 6 = 30, and writes 30 ÷ 6 = 5 or 30 ÷ 5 = 6); then in pairs, exchange papers and compare answers.
Level 5 Bridging	Writes a word problem using inverse multiplication and division (e.g.,*The teacher has 40 markers for an art project; there are 5 groups of students; how many markers does each group get? When the project is finished, the teacher collects the markers from each group. How many markers does the teacher collect?); t*hen writes the corresponding number sentences.

M (N&O):4:4	**Accurately solves problems involving** multiple operations on whole numbers or the use of the properties of factors and multiples; and addition or subtraction of decimals and positive proper fractions with like denominators. (Multiplication limited to 2 digits by 2 digits, and division limited to 1 digit divisors.)
Level 1 Entering	Matches number sentence cards showing problems involving addition or subtraction of decimals and fractions with visual representations of the problems.
Level 2 Beginning	Solves problems involving addition or subtraction of decimals or fractions, using charts or manipulatives, and states the answer orally using technical vocabulary (e.g., *tenths, hundredths, thousandths*).
Level 3 Developing	Follows teacher's written prompts to solve problems involving addition or subtraction of decimals and fractions.
Level 4 Expanding	Writes and solves word problems involving addition or subtraction of decimals and fractions, in pairs; then exchanges problems and solutions with another pair

	and compares answers.
Level 5 Bridging	Explains steps to solve a problem involving addition or subtraction of decimals and fractions, and justifies the answer in a small group discussion.

Geometry and Measurement

Geometry and the related area of measurement help students represent, describe, and make sense of the world in which they live. Geometry is also a natural place for students to develop their reasoning and justification skills.

We live in a three-dimensional world. To interpret, understand, and appreciate that world, students need to develop an understanding of space. In addition, success in mathematics depends, in part, on the development of spatial abilities. Spatial skills include making and interpreting drawings, forming mental images, and visualizing changes.

Measurement is the process of assigning a numerical value to an attribute of an object. The study of measurement provides students with techniques and tools they will need to describe and analyze their world. It also provides an opportunity to make connections within mathematics and between mathematics and other curricular areas. High school students must develop more mature insights into the essential role of measurement as a link between the abstractness of mathematics and the concreteness of the real-world.

In both areas, geometry and measurement, students need to investigate, experiment, and explore geometric properties using both technology and hands-on materials.

M (G&M):4:1	**Uses properties or attributes of angles** (number of angles) **or sides** (number of sides, length of sides, parallelism, or perpendicularity) **to identify, describe, or distinguish among** triangles, squares, rectangles, rhombi, trapezoids, hexagons, or octagons; or classify angles relative to 90° as more than, less than, or equal to.
Level 1 Entering	Matches attributes (e.g., parallel lines, number of sides) with shapes, using a word bank and pictures of everyday objects.
Level 2 Beginning	Classifies shapes according to their properties or attributes, using a multi-column table with headings such as *3 angles, 4 angles, 3 sides, 4 sides, equal sides, unequal sides, parallel sides;* students draw pictures or place cutouts of shapes in the correct columns, in a small group.
Level 3 Developing	Identifies shapes shown in a picture by their properties or attributes (e.g., number of angles, number of sides, length of sides, parallelism, perpendicularity), using a word bank.
Level 4 Expanding	Describes shapes to a partner, using their properties and/or attributes and a drawing of the shape (e.g., *A square has four equal sides and four right angles).*
Level 5 Bridging	Compares and contrasts shapes using properties and attributes (e.g., *A __ is similar to/different from a ___because…*).

M (G&M):4:3	**Uses properties or attributes** (shape of bases or number of lateral faces) **to identify, compare, or describe three-dimensional shapes** (rectangular prisms, triangular prisms, cylinders, or spheres).
Level 1 Entering	Matches cards showing attributes (e.g., *face, base, cube, sphere*) with 3-D shapes, using a word bank and pictures of everyday objects.
Level 2 Beginning	Classifies 3-D shapes according to their properties or attributes, using a multi-column table (with headings such as *rectangular base, triangular base, round base, no base, 3 lateral faces, 4 lateral faces);* students place pictures or models of solid figures in the correct columns, in a small group.

Level 3 Developing	Identifies 3-D shapes shown in a picture by their properties or attributes (e.g., shape of bases, number of lateral faces), using a word bank.
Level 4 Expanding	Describes 3-D shapes to a partner, using their properties or attributes and a drawing or model of the shape (e.g., *A triangular prism has two triangular bases and three lateral faces).*
Level 5 Bridging	Compares and contrasts 3-D shapes using properties and attributes (e.g., *A __ is similar to/different from a ___because…*).

M (G&M):4:4	**Demonstrates conceptual understanding of congruency** by matching congruent figures using reflections, translations, or rotations (flips, slides, or turns), or as the result of composing or decomposing shapes using models or explanations.
Level 1 Entering	Matches congruent shapes (manipulatives or drawings), using flips, slides, or turns.
Level 2 Beginning	Matches congruent shapes (manipulatives or drawings), with a partner, using flips, slides, or turns, and uses short phrases to describe how they are congruent.
Level 3 Developing	Matches congruent shapes (manipulatives or drawings), using flips, slides, or turns, and discusses in a small group why they are congruent.
Level 4 Expanding	Matches congruent shapes (manipulatives or drawings), using flips, slides, or turns, and writes a sentence, with a partner, explaining why they are congruent.
Level 5 Bridging	Composes a fact sentence about congruent shapes, including flips, slides, or turns, using a word bank if needed.

M (G&M):4:5	**Demonstrates conceptual understanding of similarity** by applying scales on maps, or applying characteristics of similar figures (same shape but not necessarily the same size) to identify similar figures, or to solve problems involving similar figures. Describes relationships using models or[SC] explanations.
Level 1 Entering	Matches scaled representations of similar figures, using drawings or manipulatives.
Level 2 Beginning	Describes similarities of scaled representations in single words or short phrases, using drawings or manipulatives.
Level 3 Developing	Identifies scaled figures by following a verbal prompt (e.g., *Which figure shows a 1:2 scaling of the other triangle?).*
Level 4 Expanding	Constructs or draws similar scaled figures following written prompts (e.g., instructions on a worksheet).
Level 5 Bridging	Writes a fact sentence about similar figures by describing the relationship (e.g. *This small triangle is 1/3 scale of the larger triangle).*

M (G&M):4:6	**Demonstrates conceptual understanding of perimeter of** polygons, and **the area** of rectangles, polygons or irregular shapes on grids using a variety of models, manipulatives, or formulas. Expresses all measures using appropriate units.
Level 1 Entering	Draws a diagram showing how to solve a perimeter or area problem, based on a picture of a real-world situation (e.g., how much paint to cover a wall, how much fence to go around a yard).
Level 2 Beginning	Follows simple verbal prompts to write and solve perimeter and area problems, including drawing a shape with the proper unit measurements.
Level 3 Developing	Solves perimeter and area problems, and states the answer orally using technical vocabulary (e.g., *length, width, formula, area, perimeter, units),* using a word bank, with a partner.
Level 4 Expanding	Describes the steps involved in solving perimeter and area problems, stating the entire problem and its solution, including the unit measurements of the

	shape.
Level 5 Bridging	Writes and solves a problem involving the perimeter or area of a shape, with a partner; then exchanges problems with another pair, and compares answers.

M(N&O):4:7	**Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems** across the content strands.
Level 1 Entering	Measures using metric or customary units; converts measurements within systems, given the formulas (e.g., *inches and feet; meters and kilometers*).
Level 2 Beginning	Solves measurement conversion problems, given the formulas, and following the teacher's prompts (e.g., *Measure the height of the door in inches and convert the inches to feet).*
Level 3 Developing	Converts measurements using formulas; states answers orally using technical vocabulary from a word bank (e.g., *units, inches, millimeters, kilometers*).
Level 4 Expanding	Solves measurement or conversion problems, stating the entire problem and its solution orally, including identifying the unit measurements in the problem, with a partner.
Level 5 Bridging	Creates and solves a measurement and conversion problem (e.g., student writes a fact sentence involving a classroom object and converts it into a different unit of measurement within the same system).

Functions and Algebra

Algebra is the language through which much of mathematics is communicated. Students in Kindergarten begin to explore algebraic concepts using informal representations (e.g., words, physical models, tables, graphs). In later years students progress to more abstract representations. The study of patterns is one of the central themes of algebraic thinking and leads to an understanding of relations and functions. Students at all grade-levels should recognize, describe, and generalize patterns and build mathematical models to describe, interpret, and predict the behavior of real-world phenomenon. Algebraic processes are important tools that students can use throughout their lives.

M (F&A):4:1	**Identifies and extends to specific cases a variety of patterns** (linear or nonlinear) represented in models, tables or sequences, and writes a rule in words or[SC] symbols to find the next case.
Level 1 Entering	Matches a pattern in a sequence or on a table with the next case(s), with a partner.
Level 2 Beginning	Matches a pattern in a sequence or on a table with the rule, with a partner.
Level 3 Developing	Determines, in a small group, the next case(s) from an incomplete pattern in a sequence or on a table.
Level 4 Expanding	Determines, in a small group, the next case(s) from a pattern in a sequence or on a table, and states the rule orally.
Level 5 Bridging	Determines the next case(s) from a pattern on a table or in a sequence and writes the rule.

M (F&A):4:3	**Demonstrates conceptual understanding of algebraic expressions** by using letters or symbols to represent unknown quantities to write simple linear algebraic expressions involving any one of the four operations; or by evaluating simple linear algebraic expressions using whole numbers.
Level 1 Entering	Identifies algebraic expressions from given sets of expressions; then points to variables, numerical values, and operations signs when asked, using visual supports.
Level 2	Sequences sets of cards showing algebraic expressions with a variable,

Beginning	algebraic expressions with a number substituting for the variable, and algebraic expressions with a solution.
Level 3 Developing	Writes an algebraic expression to represent an oral phrase (e.g. *4 more than a number:* student writes *x + 4; something is decreased by 5:* student writes *x – 5).*
Level 4 Expanding	Describes, with a partner, the steps used to evaluate and simplify one-step algebraic linear expressions, using technical vocabulary from a word bank (e.g., *Substitute the given number for the variable, …).*
Level 5 Bridging	Evaluates simple linear expressions given 2 or 3 values for the variable (e.g., evaluate $x + 4$ when $x = 6$ and when $x = 3$; or $x – y$ when $x = 5$ and $y = 2$), and describes the steps in writing.

M (F&A):4:4	**Demonstrates conceptual understanding of equality** by showing equivalence between two expressions using models or different representations of the expressions, by simplifying numerical expressions where left to right computations may be modified only by the use of parentheses [e.g., 14 – (2 x 5)] (expressions consistent with the parameters of M:F&A:4–3), and by solving one-step linear equations of the form $ax = c$, $x \pm b = c$, where *a, b,* and *c* are whole numbers with $a \neq 0$.
Level 1 Entering	Recognizes equality by selecting correct equations, given a set of choices (e.g., choose the correct equation from the following: 3 + 6 = 6 + 6 + 6; 3 x 6 = 6 + 6 + 6; 6 x 6 x 6 = 3 + 6; 3 + 6 = 3 x 6).
Level 2 Beginning	Generates two or more equivalent expressions for a given expression (e.g. 3 + 2 = ?, 4 + 1 + ?), with a partner.
Level 3 Developing	Solves, with a partner, one-step linear equations using manipulatives, different representations, and/or symbols (e.g., *n* pennies + 3 pennies = 7 pennies).
Level 4 Expanding	Solves a two-step algebraic equation by showing equivalence between two expressions (e.g., 4 + *n* = 2 + 5), using manipulatives, and compares answers with other students, discussing differences.
Level 5 Bridging	Solves an algebraic equation by showing equivalence between two expressions (e.g., 4 + *n* = 2 + 5), and justifies the answer in writing.

Data, Statistics, and Probability

Collecting, organizing, and displaying data, as well as interpreting and analyzing the information to make decisions and predictions, have become very important in our society. Statistical instruction should be carried out in a spirit of investigation and exploration so students can answer and formulate questions about data. Probability should be studied in familiar contexts. Students need to investigate fairness, chances of winning, and uncertainty. Technology should be used as a tool throughout the investigation process.

M (DSP):4:1	**Interprets a given representation** (line plots, tables, bar graphs, pictographs, or circle graphs) to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems. IMPORTANT: *Analyzes data consistent with concepts and skills in M:DSP:4:2.)*
Level 1 Entering	Identifies the minimum (least) and maximum (most) data points when looking at a plot, table, or graph, with a partner.
Level 2 Beginning	Describes information shown on a graph (e.g., *measure temperature*), using a word bank.
Level 3 Developing	Identifies quantities for each category on a graph (e.g., *Six students have gone to five or more states.*), with a partner.
Level 4 Expanding	Utilizes the data on a graph to solve a word problem (e.g., *How many more students went to Massachusetts than to Maine?).*

Level 5 Bridging	Summarizes the data on a graph or table to solve a word problem that involves predicting from the data, and justifies the answer orally or in writing (e.g., students conclude that data showing participation in soccer increased by 2 each school year, and predict how many there will be in four more years).

M (DSP):4:2	**Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using** measures of central tendency (median or mode), or range.
Level 1 Entering	Points to the median, mode, and range in a given set of data.
Level 2 Beginning	Matches data set cards where the median, mode, or range is identified, to vocabulary cards for those three terms, with a partner.
Level 3 Developing	Identifies measures of central tendency and range in a set of data, in a small group.
Level 4 Expanding	Interprets given data to answer questions (e.g., *What test score is most common, according to the graph?).*
Level 5 Bridging	Writes fact sentences about a set of data, using median, mode, and range.

M (DSP):4:4	**Uses counting techniques to solve problems** in context involving combinations or simple permutations (e.g., Given a map – Determine the number of paths from point A to point B) using a variety of strategies (e.g., organized lists, tables, tree diagrams or[SC] others).
Level 1 Entering	Groups drawings, manipulatives, or people to show understanding of different combinations (e.g., *There are 3 students in a group; how many different pairs can you make?),* with a partner.
Level 2 Beginning	Arranges a selection of manipulatives into as many combinations as possible (e.g., *You have four shapes, how many different combinations of two can you make?);* then records the various combinations in an organized list, table, tree diagram, or other representation, in a small group.
Level 3 Developing	Determines how many different combinations of objects can be made by using tally marks, and creates an organized list, table, tree diagram, or other representation, with a partner.
Level 4 Expanding	Prepares an oral presentation to the class, in a small group, on the number of different combinations of objects that can be made with various numbers of objects, with each member of the group creating a different representation of the combinations.
Level 5 Bridging	Creates and labels a visual representation of the problem-solving strategy used in Level 4, and writes fact sentences about the combinations that were created.

M (DSP):4:5	**For a probability event in which the sample space may or may not contain equally likely outcomes, determines** the theoretical probability of an event and expresses the result as part to whole (e.g., two out of five).
Level 1 Entering	Matches the words *less likely, likely,* and *more likely* with the appropriate color marble from a jar containing different numbers of marbles of three different colors, with a partner.
Level 2 Beginning	States orally the theoretical probability of choosing a marble of a particular color from a jar containing different numbers of marbles of three different colors, using a word bank that includes words *less likely, likely,* and *more likely.*
Level 3 Developing	Writes expressions to represent the theoretical probability of an event (e.g., *The probability of choosing a green marble out of the jar is 2 out of 5, or 2:5),* using manipulatives as needed.
Level 4 Expanding	Predicts the theoretical probability of a specific event, given three or more color choices; writes sentences to show the likelihood of choosing these colors, with

	a partner.
Level 5 Bridging	Writes and solves a probability word problem, and justifies the answer.